Meta-Analyses of Structural Equation Models in Education

Bringing together the best of two worlds

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MASEM Concepts
Meta-Analysis

$k$ studies

$r_1$
$r_2$

$\ldots$

$r_k$

Effect sizes from $k$ primary studies

$\bar{r}$

Weighted average true effect size

Moderators

Between-study heterogeneity

$x_i$

$\tau^2$
Meta-Analysis

Multiple correlations per study

Key goals
- Test the fit of a model
- Compare different models
- Explain variation in model parameters by moderators

Testing theories and complex models
Meta-Analysis

Separate univariate meta-analyses
Univariate-\( r \) approach

Aggregated correlation matrix

Path and structural equation modeling
Several issues:

- Dependencies among multiple correlations per study
- Varying sample sizes per study ⇒ overall $N$?
- Correlation matrix may be non-positive definite (issue in SEM!)
- Ignoring sampling variation across studies

(Cheung, 2015)
# Meta-Analytic Structural Equation Modeling

Meta-analysis of correlation matrices (i.e., multiple correlations from each study) or SEM parameters

<table>
<thead>
<tr>
<th>Correlation-based MASEM</th>
<th>Parameter-based MASEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Synthesize correlation matrices to an overall correlation matrix with some heterogeneity</td>
<td></td>
</tr>
<tr>
<td>- One- or two-stage MASEM</td>
<td></td>
</tr>
<tr>
<td>- Test the fit of models</td>
<td></td>
</tr>
<tr>
<td>- Stage 1: SEM for each study</td>
<td></td>
</tr>
<tr>
<td>- Stage 2: (Multivariate) meta-analysis of SEM parameters</td>
<td></td>
</tr>
<tr>
<td>- Quantify heterogeneity in SEM parameters</td>
<td></td>
</tr>
</tbody>
</table>

(Cheung, 2015)
Correlation-Based MASEM

Aggregating correlation matrices via multivariate meta-analysis

Structural equation modeling

Pooled correlation matrix

Correlation-Based MASEM

$r_1$, $r_2$, ..., $r_k$
Correlation-Based MASEM

<table>
<thead>
<tr>
<th>Study</th>
<th>$r_{21}$</th>
<th>$r_{31}$</th>
<th>$r_{41}$</th>
<th>$r_{32}$</th>
<th>$r_{42}$</th>
<th>$r_{43}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study 1</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Study 2</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study 3</td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Study 4</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study $k$</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Data: Correlation matrices from primary studies
Correlation-Based MASEM

Stage 1

Pooling correlation matrices: Maximum likelihood estimation

Pooled correlations and variance components:

\[ \mu(P) = (\rho_{21}, \rho_{31}, \rho_{41}, \rho_{32}, \rho_{42}, \rho_{43}) \]

\[ \Sigma(P) = V_i + T^2 \]

\textit{Acov}: Asymptotic covariance matrix of pooled correlations
Correlation-Based MASEM

Stage 2

Pooled correlation matrix:

<table>
<thead>
<tr>
<th></th>
<th>$\rho_{21}$</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{31}$</td>
<td>$\rho_{32}$</td>
<td>1.00</td>
</tr>
<tr>
<td>$\rho_{41}$</td>
<td>$\rho_{42}$</td>
<td>$\rho_{43}$</td>
</tr>
</tbody>
</table>

Pooled correlations as observed input matrix

SEM through weighted least squares (WLS) estimation
Parameter-Based MASEM

$r_1$, $r_2$, ..., $r_k$

Structural equation modeling for each study

Uni- or multivariate meta-analysis

Pooled model parameters

$\beta_1, ..., \beta_k$

$\lambda_1, ..., \lambda_k$
Parameter-Based MASEM

Stage 1

Perform SEM to each study $\rho_i = \rho(\theta_i)$

Model parameters and variance components:

Parameter vectors $t_i = \hat{\theta}_i$
Asymptotic covariance matrix $V_{ti}$
Parameter-Based MASEM

Uni- or multivariate meta-analysis with random effects:

\[ t_i = \theta_R + u_{\theta i} + e_{\theta i} \]

\[ T_{\theta}^2 = Cov(u_{\theta i}) \]

\[ V_{t_i} = Cov(e_{\theta i}) \]

Stage 2

Perform meta-analysis on the model parameters
(Fixed-, random-, and mixed-effects models)

Moderation of model parameters
Multilevel data structures
MASEM Example
Technology Acceptance Model (TAM)

A Model Explaining Teachers’ Intentions to Use Technology

Variables
- BI: Intentions to use technology, ATT: Attitudes toward technology
- PEU: Perceived ease of use, PU: Perceived usefulness

Model 1

Model 2
Meta-analytic data set based on teacher samples

Data set
- 50 studies with independent samples
- 4 variables
- 300 correlations
- 50 correlation matrices
- Pre- and in-service teacher samples

Main RQs
1. **Model evaluation**: To what extent does the TAM represent the teacher data across samples?
2. **Model comparison**: To what extent does a direct effect exist?

⇒ Correlation-based MASEM
## Technology Acceptance Model (TAM)

Meta-analytic data set based on teacher samples

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>AA</th>
<th>AB</th>
<th>AC</th>
<th>AD</th>
<th>AE</th>
<th>AF</th>
<th>AG</th>
<th>AH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>StudyID</td>
<td>Sample size</td>
<td>PU-PU</td>
<td>PU-PEOU</td>
<td>PEOU</td>
<td>PU-ATT</td>
<td>PEOU-ATT</td>
<td>ATT-ATT</td>
<td>PU-BI</td>
<td>PEOU-BI</td>
<td>ATT-BI</td>
<td>BI-BI</td>
</tr>
<tr>
<td>2</td>
<td>217</td>
<td>446</td>
<td>1</td>
<td>0,61</td>
<td>1</td>
<td>0,6776</td>
<td>0,5138</td>
<td>1</td>
<td>0,599184</td>
<td>0,449892</td>
<td>0,860328</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>247</td>
<td>88</td>
<td>1</td>
<td>0,661</td>
<td>1</td>
<td>0,59</td>
<td>0,584</td>
<td>1</td>
<td>0,75</td>
<td>0,746</td>
<td>0,684</td>
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</tr>
<tr>
<td>4</td>
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<td>1</td>
<td>0,423</td>
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<td>0,374</td>
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<td>0,512</td>
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</tr>
<tr>
<td>5</td>
<td>289</td>
<td>234</td>
<td>1</td>
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<td>1</td>
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<td>0,55</td>
<td>1</td>
<td>0,55</td>
<td>0,48</td>
<td>0,59</td>
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<td>6</td>
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<td>29</td>
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<td>0,461</td>
<td>0,754</td>
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<td>0,364</td>
<td>0,669</td>
<td>0,595</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>320</td>
<td>313</td>
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<td>0,279</td>
<td>1</td>
<td>0,63</td>
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<td>0,319</td>
<td>0,441</td>
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<tr>
<td>8</td>
<td>332A</td>
<td>169</td>
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<td>0,68</td>
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<td>0,68</td>
<td>0,73</td>
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<td>0,34</td>
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<td>170</td>
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<td>1</td>
<td>0,56</td>
<td>0,43</td>
<td>1</td>
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<td>0,32</td>
<td>0,36</td>
<td>1</td>
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<tr>
<td>10</td>
<td>335A</td>
<td>151</td>
<td>1</td>
<td>0,329</td>
<td>1</td>
<td>0,565</td>
<td>0,349</td>
<td>1</td>
<td>0,523</td>
<td>0,327</td>
<td>0,462</td>
<td>1</td>
</tr>
</tbody>
</table>
Step 0
Check correlation matrices for their positive definiteness

# PD check
pd_check <- is.pd(TAM$data)
sum(!is.na(pd_check))

## [1] 50

pd_check

## Okazaki & Renda dos Santos (2012)  TRUE
## Wong (2015)                         TRUE
Step 1
Pooling correlation matrices across independent samples
- REM
- Diagonal constraints of the heterogeneity matrix

# Combine correlation matrices using a REM
#
rem <- tssem1(TAMu$data, TAMu$n,
              method = "REM",
              RE.type = "Diag")

# Summarize the results
summary(rem)
## Technology Acceptance Model (TAM)

### Step 1
Pooling correlation matrices across independent samples
- REM
- Diagonal constraints of the heterogeneity matrix

### Table

|             | Estimate | Std.Error | lbound | ubound | z value | Pr(>|z|) |
|-------------|----------|-----------|--------|--------|---------|----------|
| Intercept1  | 0.4828802| 0.0217706 | 0.4402105| 0.5255498| 22.1803 | < 2.2e-16*** |
| Intercept2  | 0.6068160| 0.0174683 | 0.5725787| 0.6410533| 34.7381 | < 2.2e-16*** |
| Intercept3  | 0.5153033| 0.0254332 | 0.4654551| 0.5651514| 20.2611 | < 2.2e-16*** |
| Intercept4  | 0.5276429| 0.0209277 | 0.4866253| 0.5686604| 25.2127 | < 2.2e-16*** |
| Intercept5  | 0.4089908| 0.0220210 | 0.3658303| 0.4521512| 18.5727 | < 2.2e-16*** |
| Intercept6  | 0.5187087| 0.0259696 | 0.4678093| 0.5696081| 19.9737 | < 2.2e-16*** |

---

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Step 1
Pooling correlation matrices across independent samples
- REM
- Diagonal constraints of the heterogeneity matrix

## Q statistic on the homogeneity of effect sizes: 3622.948
## Degrees of freedom of the Q statistic: 294
## P value of the Q statistic: 0

## Heterogeneity indices (based on the estimated Tau2):

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept1: I2 (Q statistic)</td>
<td>0.9178</td>
</tr>
<tr>
<td>Intercept2: I2 (Q statistic)</td>
<td>0.9146</td>
</tr>
<tr>
<td>Intercept3: I2 (Q statistic)</td>
<td>0.9460</td>
</tr>
<tr>
<td>Intercept4: I2 (Q statistic)</td>
<td>0.9217</td>
</tr>
<tr>
<td>Intercept5: I2 (Q statistic)</td>
<td>0.9015</td>
</tr>
<tr>
<td>Intercept6: I2 (Q statistic)</td>
<td>0.9493</td>
</tr>
</tbody>
</table>
Step 1
Pooling correlation matrices across independent samples
- REM
- Diagonal constraints of the heterogeneity matrix

```r
# Extract the pooled correlation matrix
corr.rem <- vec2symMat(coef(rem, select= "fixed"), diag=FALSE)
colnames(corr.rem) <- c("PU", "PEU", "ATT", "BI")
rownames(corr.rem) <- c("PU", "PEU", "ATT", "BI")
corr.rem
```

```r
##          PU     PEU      ATT      BI
## PU 1.0000000 0.4828802 0.6068160 0.5153033
## PEU 0.4828802 1.0000000 0.5276429 0.4089908
## ATT 0.6068160 0.5276429 1.0000000 0.5187087
## BI  0.5153033 0.4089908 0.5187087 1.0000000
```
Step 2
Model specification
- Lavaan syntax possible
- Translate into the RAM framework

# Model specification
Model1 <- " # Structural parameters
  BI ~ ATT + PU
  ATT ~ PEU + PU
  PU ~ PEU

  # Variance of independent variables
  PEU ~ 1*PEU"

"
# Convert the model into the RAM language
RAM <- lavaan2RAM(Model1,
                   obs.variables = c("PU", "PEU", "ATT", "BI"))

# Declare the matrices in the RAM framework
# A: Matrix of factor loadings and regression coefficients
A <- RAM$A
# S: Matrix of variances and covariances
S <- RAM$S
# F: Matrix indicating manifest or latent variables (0=latent, 1=manifest)
F <- RAM$F

# Check the RAM specification
# Note: The function remains silent if the specification is correct.
checkRAM(A, S, cor.analysis = TRUE)
Technology Acceptance Model (TAM)

Step 2
Model estimation
- Pooled correlation matrix and model-specifying matrices as input
- Diagonal constraints

# Model estimation
# Confidence intervals are likelihood-ratio based (LBCIs)
TS.Model1 <- tssem2(rem, Amatrix=A, Smatrix=S, Fmatrix=F,
                   intervals.type="LB",
                   diag.constraints = TRUE,
                   model.name = "Model 1 with the direct effect")
Step 2
Model estimation
- Pooled correlation matrix and model-specifying matrices as input
- Diagonal constraints

# Summarize the model parameters
summary(TS.Model1)

## 95% confidence intervals: Likelihood-based statistic
## Coefficients:

|        | Estimate | Std.Error | lbound | ubound | z value | Pr(>|z|) |
|--------|----------|-----------|--------|--------|---------|----------|
| ATONPEU| 0.32668  | NA        | 0.26635| 0.38442| NA      | NA       |
| ATONPU | 0.43668  | NA        | 0.38557| 0.48754| NA      | NA       |
| BIONATT| 0.35455  | NA        | 0.26355| 0.44470| NA      | NA       |
| BIONPU | 0.32253  | NA        | 0.23013| 0.41347| NA      | NA       |
| PUONPEU| 0.49973  | NA        | 0.45797| 0.54132| NA      | NA       |
| ATTWITHATT| 0.56001| NA        | 0.51662| 0.60096| NA      | NA       |
| BIWITHBI| 0.63306  | NA        | 0.58739| 0.67590| NA      | NA       |
| PWITHPU| 0.75027  | NA        | 0.70697| 0.79026| NA      | NA       |
Step 2
Model estimation
- Pooled correlation matrix and model-specifying matrices as input
- Diagonal constraints

```r
# Summarize the model parameters
summary(TS.Model1)
```

## Goodness-of-fit indices:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample size</td>
<td>14918.000</td>
</tr>
<tr>
<td>Chi-square of target model</td>
<td>10.0900</td>
</tr>
<tr>
<td>DF of target model</td>
<td>1.0000</td>
</tr>
<tr>
<td>p value of target model</td>
<td>0.0015</td>
</tr>
<tr>
<td>Number of constraints imposed on &quot;Smatrix&quot;</td>
<td>3.0000</td>
</tr>
<tr>
<td>DF manually adjusted</td>
<td>0.0000</td>
</tr>
<tr>
<td>Chi-square of independence model</td>
<td>3002.8440</td>
</tr>
<tr>
<td>DF of independence model</td>
<td>6.0000</td>
</tr>
<tr>
<td>RMSEA</td>
<td>0.0247</td>
</tr>
<tr>
<td>RMSEA lower 95% CI</td>
<td>0.0125</td>
</tr>
<tr>
<td>RMSEA upper 95% CI</td>
<td>0.0395</td>
</tr>
<tr>
<td>SRMR</td>
<td>0.0285</td>
</tr>
<tr>
<td>TLI</td>
<td>0.9818</td>
</tr>
<tr>
<td>CFI</td>
<td>0.9970</td>
</tr>
<tr>
<td>AIC</td>
<td>8.0900</td>
</tr>
<tr>
<td>BIC</td>
<td>0.4797</td>
</tr>
</tbody>
</table>
Technology Acceptance Model (TAM)

Step 2
Path diagram

# Visualize the model and add its parameters
TS.Model1.plot <- meta2semPlot(TS.Model1)
semPaths(TS.Model1.plot, whatLabels="est",
        rotation=2,
        edge.label.cex = 1.25,
        sizeMan = 8,
        color = "grey",
        layout = "tree2")
Technology Acceptance Model (TAM)

Step 3
Model comparison

# Model comparison
anova(TS.Model1, TS.Model2)

Likelihood-ratio test: $\chi^2(1) = 42.1, p < .001$
Technology Acceptance Model (TAM)

Meta-analytic data set based on teacher samples

Data set
- 50 studies with independent samples
- 5 structural parameters per study (path coefficients)
- 250 structural parameters
- Pre- and in-service teacher samples

Main RQs

3. **Model parameters:** To what extent do the structural parameters in the TAM vary between studies?

4. **Moderators:** Which study characteristics explain this possible variation?

⇒ Parameter-based MASEM
**Step 1**

Structural equation modeling of the correlation matrices for each study

- Structural parameters
- Asymptotic covariance matrix
- Model fit

---

```
## [[1]]$coefs
##  att2bi  pu2bi  pu2att  peu2att  peu2pu
##     0.8400581  0.03000572  0.16013217  0.58007042  0.61042985
##
## [[1]]$vcoefs
##  att2bi  pu2bi  pu2att  peu2att  peu2pu
##  att2bi  1.0750986e-03  -7.286343e-04  4.264239e-19  -1.001616e-18  1.074360e-19
##  pu2att  8.232048e-19  1.827367e-20  1.871300e-03  -1.143102e-03  2.263327e-18
##  peu2att  -1.009124e-18  4.188215e-19  -1.143102e-03  1.873938e-03  -1.300830e-18
##  peu2pu  2.395128e-19  -3.614278e-19  2.329496e-19  1.507669e-18  1.404691e-03

## [[2]]$coefs
##  att2bi  pu2bi  pu2att  peu2att  peu2pu
##     0.3704956  0.5317801  0.3458485  0.3624505  0.6631249
##
## [[2]]$vcoefs
##  att2bi  pu2bi  pu2att  peu2att  peu2pu
##  pu2bi  -3.585940e-03  6.056146e-03  2.673442e-18  -1.136794e-17  -4.247357e-19
##  pu2att  -2.308345e-18  2.535713e-18  1.173156e-02  -7.804501e-03  -9.524999e-19
##  peu2att  5.721964e-18  -7.748799e-18  -7.804501e-03  1.180711e-02  6.357375e-18
```
## Technology Acceptance Model (TAM)

Model fit indices:

<table>
<thead>
<tr>
<th>Model</th>
<th>chisq</th>
<th>pvalue</th>
<th>ntotal</th>
<th>cfi</th>
<th>tli</th>
<th>rmsea</th>
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<tbody>
<tr>
<td>Okazaki &amp; Renda dos Santos (2012)</td>
<td>0.0011</td>
<td>0.9994</td>
<td>446</td>
<td>1.0000</td>
<td>1.0055</td>
<td>0.0000</td>
</tr>
<tr>
<td>Parkman (2015)</td>
<td>17.5677</td>
<td>0.0002</td>
<td>88</td>
<td>0.9230</td>
<td>0.7690</td>
<td>0.2974</td>
</tr>
<tr>
<td>Wong (2015)</td>
<td>7.5486</td>
<td>0.0230</td>
<td>234</td>
<td>0.9837</td>
<td>0.9512</td>
<td>0.1089</td>
</tr>
<tr>
<td>Cote &amp; Miliner (2015)</td>
<td>5.1095</td>
<td>0.0777</td>
<td>29</td>
<td>0.9323</td>
<td>0.7969</td>
<td>0.2315</td>
</tr>
<tr>
<td>Teo &amp; Milutinovic (2015)</td>
<td>0.0026</td>
<td>0.9987</td>
<td>313</td>
<td>1.0000</td>
<td>1.0065</td>
<td>0.0000</td>
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<tr>
<td>Teo &amp; Fan et al. (2015)-1</td>
<td>0.5738</td>
<td>0.7506</td>
<td>169</td>
<td>1.0000</td>
<td>1.0156</td>
<td>0.0000</td>
</tr>
<tr>
<td>Teo &amp; Fan et al. (2015)-2</td>
<td>6.4464</td>
<td>0.0398</td>
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<td>0.9660</td>
<td>0.8981</td>
<td>0.1144</td>
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<tr>
<td>Koutrmanos et al. (2015)-1</td>
<td>3.4601</td>
<td>0.1773</td>
<td>151</td>
<td>0.9893</td>
<td>0.9679</td>
<td>0.0695</td>
</tr>
<tr>
<td>Koutrmanos et al. (2015)-2</td>
<td>0.1877</td>
<td>0.9104</td>
<td>106</td>
<td>1.0000</td>
<td>1.0644</td>
<td>0.0000</td>
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<tr>
<td>Wu et al. (2015)</td>
<td>4.0770</td>
<td>0.1302</td>
<td>340</td>
<td>0.9963</td>
<td>0.9890</td>
<td>0.0553</td>
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<tr>
<td>Kabakci-Yurdakul et al. (2014)</td>
<td>3.8237</td>
<td>0.1478</td>
<td>579</td>
<td>0.9988</td>
<td>0.9963</td>
<td>0.0397</td>
</tr>
<tr>
<td>Kirmizi (2014)</td>
<td>1.4549</td>
<td>0.4831</td>
<td>213</td>
<td>1.0000</td>
<td>1.0042</td>
<td>0.0000</td>
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<tr>
<td>Mac Callum et al. (2014)</td>
<td>1.5213</td>
<td>0.4674</td>
<td>196</td>
<td>1.0000</td>
<td>1.0278</td>
<td>0.0000</td>
</tr>
<tr>
<td>Wong et al. (2013)</td>
<td>2.2451</td>
<td>0.3255</td>
<td>302</td>
<td>0.9993</td>
<td>0.9980</td>
<td>0.0201</td>
</tr>
</tbody>
</table>
Technology Acceptance Model (TAM)

**Step 1**
Structural equation modeling of the correlation matrices for each study
- Structural parameters
- Asymptotic covariance matrix
- Model fit

Variance explanation:

<table>
<thead>
<tr>
<th></th>
<th>BI</th>
<th>ATT</th>
<th>PU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Okazaki &amp; Renda dos Santos (2012)</td>
<td>0.7408</td>
<td>0.4755</td>
<td>0.3726</td>
</tr>
<tr>
<td>Parkman (2015)</td>
<td>0.6532</td>
<td>0.4172</td>
<td>0.4397</td>
</tr>
<tr>
<td>Wong (2015)</td>
<td>0.3988</td>
<td>0.4808</td>
<td>0.2508</td>
</tr>
<tr>
<td>Cote &amp; Miliner (2015)</td>
<td>0.3692</td>
<td>0.5846</td>
<td>0.2690</td>
</tr>
<tr>
<td>Teo &amp; Milutinovic (2015)</td>
<td>0.2555</td>
<td>0.9242</td>
<td>0.0781</td>
</tr>
<tr>
<td>Teo &amp; Fan et al. (2015)-1</td>
<td>0.1248</td>
<td>0.5969</td>
<td>0.4639</td>
</tr>
<tr>
<td>Teo &amp; Fan et al. (2015)-2</td>
<td>0.1309</td>
<td>0.3648</td>
<td>0.1608</td>
</tr>
<tr>
<td>Koutromanos et al. (2015)-1</td>
<td>0.3145</td>
<td>0.3496</td>
<td>0.1089</td>
</tr>
</tbody>
</table>
Step 2
Multivariate meta-analysis

# Specify the random-effects model
# Use a diagonal matrix due to limited sample size
M1_random <- meta(tra_coefs_all,
    tra_vcoefs_all,
    RE.constraints = Diag(paste("0.2*Tau2","1:5","_",1:5, sep="")),
    model.name = "Multivariate REM"
)

# Summarize the results
summary(M1_random)
Technology Acceptance Model (TAM)

## 95% confidence intervals: z statistic approximation (robust=FALSE)
## Coefficients:
##        Estimate  Std.Error  lbound  ubound  z value Pr(>|z|)
## Intercept1  0.3385985  0.0283137  0.2831047  0.3940923  11.9588 < 2.2e-16 ***
## Intercept2  0.3034082  0.0297130  0.2451718  0.3616446  10.2113 < 2.2e-16 ***
## Intercept3  0.3044215  0.0232637  0.2588254  0.3500175  13.0857 < 2.2e-16 ***
## Intercept4  0.4650990  0.0189385  0.4279801  0.5022178  24.5583 < 2.2e-16 ***
## Intercept5  0.4875640  0.0230803  0.4423275  0.5328006  21.1247 < 2.2e-16 ***
## Tau2_1_1    0.0353438  0.0077820  0.0200913  0.0505964  4.5417  5.580e-06 ***
## Tau2_2_2    0.0393800  0.0087935  0.0221451  0.0566149  4.4783  7.523e-06 ***
## Tau2_3_3    0.0236959  0.0054885  0.0129387  0.0344532  4.3174  1.579e-05 ***
## Tau2_4_4    0.0146157  0.0036370  0.0074873  0.0217441  4.0186  5.854e-05 ***
## Tau2_5_5    0.0229411  0.0053725  0.0124113  0.0334710  4.2701  1.954e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Technology Acceptance Model (TAM)

Step 2
Multivariate meta-analysis

## Q statistic on the homogeneity of effect sizes: 3375.975
## Degrees of freedom of the Q statistic: 245
## P value of the Q statistic: 0
##
## Heterogeneity indices (based on the estimated Tau2):
##
## | Intercept1: I2 (Q statistic) | Estimate |
## |-------------------------------|----------|
## | I2 (Q statistic)              | 0.9224   |
## | I2 (Q statistic)              | 0.9298   |
## | I2 (Q statistic)              | 0.9214   |
## | I2 (Q statistic)              | 0.8782   |
## | I2 (Q statistic)              | 0.9074   |
Technology Acceptance Model (TAM)

Step 2
Multivariate meta-analysis with possible moderators

```r
# Specify the mixed-effects model
# Use a diagonal matrix due to limited sample size
M1_mixed <- meta(tra_coefs_all,
                  tra_vcoefs_all,
                  RE.constraints = Diag(paste("0.2*Tau2_, 1:5, _", 1:5, sep="")),
                  model.name = "Multivariate MEM",
                  x = cbind(TS, Technology))

# Summarize the results
summary(M1_mixed)
```
### Step 2
Multivariate meta-analysis with possible moderators

| # | Estimate  | Std. Error | lbound  | ubound  | z value | Pr(>|z|) |
|---|-----------|------------|---------|---------|---------|----------|
| Slope1_1 | 0.1186281 | 0.0638648  | -0.0065446 | 0.2438009 | 1.8575  | 0.06324 . |
| Slope2_1 | -0.0576406 | 0.0681175  | -0.1911485 | 0.0758673 | -0.8462 | 0.39744  |
| Slope3_1 | -0.1215178 | 0.0513254  | -0.2221139 | -0.0209218 | -2.3676 | 0.01790 * |
| Slope4_1 | 0.0296254  | 0.0431604  | -0.0549674 | 0.1142183 | 0.6864  | 0.49246  |
| Slope5_1 | 0.0251933  | 0.0546888  | -0.0819948 | 0.1323813 | 0.4607  | 0.64504  |
| Slope1_2 | -0.1184871 | 0.0653974  | -0.2466635 | 0.0096894 | -1.8118 | 0.07002 . |
| Slope2_2 | 0.1215357  | 0.0697041  | -0.0150819 | 0.2581533 | 1.7436  | 0.08123 . |
| Slope3_2 | 0.0220181  | 0.0524786  | -0.0808382 | 0.1248743 | 0.4196  | 0.67481  |
| Slope4_2 | 0.0303920  | 0.0442073  | -0.0562526 | 0.1170367 | 0.6875  | 0.49177  |
| Slope5_2 | -0.0250340 | 0.0561231  | -0.1350333 | 0.0849654 | -0.4461 | 0.65556  |
Technology Acceptance Model (TAM)

Step 2
Multivariate meta-analysis with possible moderators

Variance explanation as the reduction of the between-study heterogeneity

```r
## Explained variances (R2):
##
## y1    y2    y3    y4    y5
## Tau2 (no predictor) 0.037092 0.042075 0.027398 0.016383 0.0224  
## Tau2 (with predictors) 0.032183 0.037254 0.020655 0.013689 0.0227  
## R2      0.132345 0.114586 0.246118 0.164481 0.0000
```
Meta-Analysis

Key potential
- Test and compare models representing substantive theory
- Examine heterogeneity in model parameters

Key challenges
- Multilevel structure of the primary and secondary data
- Heterogeneity and missing data
The End.
References


Two-Stage Correlation-based MASEM (TSSEM)

Stage 1: Pooling correlation matrices (multi-group SEM)
Fixed-effects model

Correlation matrix of study $i$:

\[ \Sigma_i = D_i P_i D_i \]

Population covariance matrix
Correlation matrix
Diagonal matrix with entries close to an identity matrix

Handling missing correlations through ML if data are MCAR or MAR

(Cheung, 2015; Jak & Cheung, 2018)
Stage 1: Pooling correlation matrices (multi-group SEM)

Fixed-effects model

(Cheung, 2015; Jak & Cheung, 2018)

Vector of pooled correlation matrix

\[ \hat{\rho}_F = \text{vechs}(\hat{P}_F) \]

Asymptotic sampling covariance matrix

\[ \hat{V}_F = \text{Cov}(\hat{\rho}_F) \]
Two-Stage Correlation-based MASEM (TSSEM)

Stage 2: Structural equation modeling
Fixed-effects model

(Cheung, 2015; Jak & Cheung, 2018)

Weighted least squares estimation to fit SEM
Weighting correlation elements by the inverse of the asymptotic covariance matrix
(accounting for the precision of estimates from stage 1)

\[ F_{WLS}(\theta) = (r_F - \rho_F(\theta))^T V_F^{-1} (r_F - \rho_F(\theta)) \]

Evaluation of goodness-of-fit
Subgroup analysis
Two-Stage Correlation-based MASEM (TSSEM)

Stage 1: Pooling correlation matrices (multi-group SEM)
Random-effects model

(Cheung, 2015; Jak & Cheung, 2018)

Level 1:
\[ r_i = \rho_i + e_i \]

Level 2:
\[ \rho_i = \rho_R + u_i \]

\[ e_i \sim N(0, V_i) \text{ and } u_i \sim N(0, T^2) \]

Estimation steps:
1. Sampling covariance matrix \( V_i \) for each study
2. Multivariate meta-analysis on the correlation vectors

Sampling and heterogeneity covariance matrices
Two-Stage Correlation-based MASEM (TSSEM)

Stage 1: Pooling correlation matrices (multi-group SEM)
Random-effects model

(Chemung, 2015; Chemung & Cheung, 2016)

Level 1:
\[ r_i = \rho_i + e_i \]

Level 2:
\[ \rho_i = \rho_R + u_i \]

Vector of the pooled correlation matrix \( \hat{\rho}_R \)

Asymptotic sampling covariance matrix \( \hat{V}_R \)

Heterogeneity of correlation vectors \( \hat{T}^2 \)

\[ e_i \sim N(0, V_i) \] and \( u_i \sim N(0, T^2) \)

Sampling and heterogeneity covariance matrices
Two-Stage Correlation-based MASEM (TSSEM)

Stage 2: Structural equation modeling
Random-effects model

(Cheung, 2015; Cheung & Cheung, 2016)

Weighted least squares estimation to fit SEM
Weighting correlation elements by the inverse of the asymptotic covariance matrix
(accounting for the precision of estimates from stage 1)

\[ F_{WLS}(\theta) = (r_R - \rho_R(\theta))^T V_R^{-1} (r_R - \rho_R(\theta)) \]

Note: Random effects \( \hat{T}^2 \) are not involved in the stage-2 analysis.
Full-Information MASEM (FIMASEM)

Stage 2: Structural equation modeling
Random-effects model
(Jak & Cheung, 2018; Yu et al., 2016)

Steps

1. Estimating pooled correlations and variances
2. Parametric bootstrapping ⇒ many heterogeneous correlation matrices
3. Fitting structural equation models

Quantify stage-2 heterogeneity of model parameters (credible intervals)
Full-Information MASEM (FIMASEM)

Stage 2: Structural equation modeling
Random-effects model

(Jak & Cheung, 2018; Cheung, 2018)

Issues

- Correct credible intervals but incorrect test statistics and goodness-of-fit indices ⇒ Model fit evaluation compromised
- No moderators to explain between-study variation possible

Quantify stage-2 heterogeneity of model parameters (credible intervals)